General Certificate of Education
January 2003
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS (SPECIFICATION A) MAP5
Unit Pure 5

Friday 17 January 2003 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.
- Sheets of graph paper are available on request.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

P60387/0103/MAP5 6/6/1050

Answer **all** questions.

1 (a) Write down the value of $\lim\limits_{x \to \infty} x^k e^{-4x}$, where k is a positive constant. *(1 mark)*

(b) Show that $\displaystyle\int_0^{\infty} x e^{-4x} \mathrm{d}x = \frac{1}{16}$. *(6 marks)*

(c) Hence find $\displaystyle\int_0^{\infty} x^2 e^{-4x} \mathrm{d}x$. *(3 marks)*

2 The polar equation of a curve C is

$$r = \frac{1}{\theta}, \quad \frac{\pi}{4} \leqslant \theta \leqslant 2\pi.$$

(a) Sketch C. *(3 marks)*

(b) The point A on C is the point where $\theta = \dfrac{\pi}{4}$ and the point B on C is the point where $\theta = 2\pi$. The point O is the pole.

(i) Given that P is the point on C where $\theta = \alpha$, show that the area of the region bounded by the curve C and the lines OA and OP is

$$\frac{2}{\pi} - \frac{1}{2\alpha}.$$

(4 marks)

(ii) Hence find the value of α for which OP bisects the area between the curve C and the lines OA and OB. *(3 marks)*

3 (a) Explain why

$$\int_{-2}^{2} \frac{\mathrm{d}x}{\sqrt{4 - x^2}}$$

is an improper integral. *(1 mark)*

(b) Evaluate this integral. *(3 marks)*

4 (a) (i) Differentiate implicitly $(x^2+1)y$ with respect to x. *(2 marks)*

(ii) Find the general solution of the differential equation

$$(x^2+1)\frac{\mathrm{d}y}{\mathrm{d}x}+2xy=6x^2+2.$$

Express your answer in the form $y=\mathrm{f}(x)$. *(4 marks)*

(b) Use the substitution $y=\frac{\mathrm{d}z}{\mathrm{d}x}$ to solve the differential equation

$$(x^2+1)\frac{\mathrm{d}^2z}{\mathrm{d}x^2}+2x\frac{\mathrm{d}z}{\mathrm{d}x}=6x^2+2,$$

given that $z=2$ and $\frac{\mathrm{d}z}{\mathrm{d}x}=1$ when $x=0$. Express your answer in the form $z=\mathrm{f}(x)$. *(7 marks)*

5 (a) Obtain the roots of the equation

$$m^2+2m+5=0$$

in the form $a+\mathrm{i}b$. *(2 marks)*

(b) Solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\frac{\mathrm{d}y}{\mathrm{d}x}+5y=10,$$

given that $y=0$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ when $x=0$. *(8 marks)*

TURN OVER FOR THE NEXT QUESTION

6 The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y),$$

where $$\mathrm{f}(x, y) = \frac{xy}{\sqrt{x^2 + y^2}}$$

and $$y(1) = 0.5.$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\,\mathrm{f}(x_r, y_r),$$

with $h = 0.1$ to obtain an approximation to $y(1.1)$ giving your answer to **four** decimal places. *(3 marks)*

(b) (i) Use the formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2),$$

where $k_1 = h\,\mathrm{f}(x_r, y_r)$

and $k_2 = h\,\mathrm{f}(x_r + h, y_r + k_1)$,

with $h = 0.1$ to obtain a further approximation to $y(1.1)$. *(5 marks)*

(ii) Use the formula given in part (b)(i), together with your value for $y(1.1)$ obtained in part (b)(i), to obtain an approximation to $y(1.2)$, giving your answer to three decimal places. *(5 marks)*

END OF QUESTIONS